Test Statistic

Hypothesis Testing - testing whether or not a statement about the unknown parameters or features of a data set is true.
Testing if your hypothesis is true or false

Null Hypothesis (h_0) - your original + initial hypothesis
It's either true or false
accepted rejected

Alternative Hypothesis (h_a) - If Ho is Rejected, then your alternative Hypotheses will be accepted.
Usually the opposite of Null Hypothesis

Ex

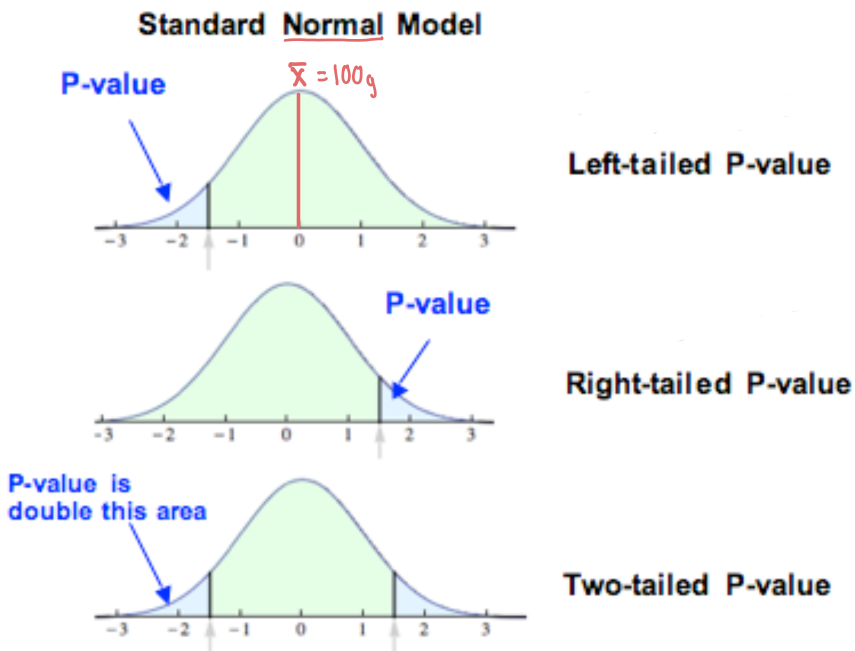
Null Hypothesis: I'm going to win $1000
Alternate Hypothesis: I'm going to win more Tyson $1000

Null Hypothesis: Justin Bieber will marry Senlena Gomez
Alternate Hypothesis:

Null Hypothesis: President Obama will be re-elected with 5% majority
Alternative Hypothesis: President Obama will be re-elected with 10% majority

? How do we know if our Hypothesis is Accepted or Rejected ex) Studying the weight of bananas with a claimed mean of 100g

H_0 $\bar{x} = 100g$



Left-Tailed Hypothesis Test - a test where the Rejection Region is located to the extreme Left of the distribution

$H_a: \bar{x} < 100g$

Right-Tailed Hypothesis Test - a test where the Rejection Region is located to the extreme Right of the distribution

$H_a: \bar{x} > 100g$

Two-Tailed Hypothesis Test - a test where the Rejection Region is divided equally between 2 Critical Values

$H_a: \bar{x} \neq 100g$

P-Value Probability of obtaining a sample more extreme than the one's observed in your data, assuming H_0 is true

χ^2 Goodness of Fit Test

χ - "Chi" [k-eye]

★ Criteria to allow us to use this test:
1. Random Sampling
2. Large Counts Condition: expected values > 5
3. Categorical variables: qualitative data

ex| Consider a test with 4 answer choices each for 100 questions. We want to see how evenly distributed the correct answer choices are amongst the 4 possible choices:

A B C D

(a) What is your Null Hypothesis?

A, B, C, D each make up 25% of correct answers

(b) What is your Alternate Hypothesis?

Not equal distribution

(c) Find χ^2

Correct Choice	Expected	Actual
A	25	20
B	25	20
C	25	25
D	25	35

★ You'll use your calculator

$$\boxed{\chi^2 = \sum \frac{(\text{Actual} - \text{Expected})^2}{\text{Expected}}}$$

$$\chi^2 = \frac{(25-20)^2}{25} + \frac{(25-20)^2}{25} + \frac{(25-25)^2}{25} + \frac{(35-25)^2}{25} = \boxed{6}$$

? O.K. so our χ^2 Test Statistic is 6

but how does this help us accept/Reject our Hypothesis

Degrees of Freedom (df) - the amount of values in a system that are free to vary = # of choices - 1

* If there are 9 positions on a softball team, then we have 8 df because once 8 people are assigned a position, the last person's position is fixed

A, B, C, D → 4 answer choices ⇒ $\boxed{df = 4 - 1 = 3}$

Significance Level (α) - The probability of "Accidentally" Rejecting H_0 when it is true.

α typically = 0.05 or 0.01

* a threshold that defines the strength of evidence

* It is a value we, as researchers choose ... depending on how important it is that our statistical analysis is precise and accurate

= 1%, 5%, or 10%

= 0.01, 0.05, 0.1

* The higher this value ⇒ the weaker the evidence

1% → very strong evidence (life or death)
5% → strong evidence + used most often
10% → moderate evidence

* α = 1 - Confidence Level

If a researcher wants to be 95% confident in the strength + accuracy of their results, then 1 - 0.95 = 0.05 = 5% significance Level

Circling back to our Multiple Choice Test Ex

We have: $\chi^2 = 6$

$df = 3$

$\alpha = 0.05$ (chosen not calculated)

Now, with all these values, we consult a Critical Value Table

α

df

Critical Value

Critical values of the Chi-square distribution with *d* degrees of freedom

	Probability of exceeding the critical value						
d	0.05	0.01	0.001	*d*	0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	$\boxed{7.815}$	11.345	16.266	13	22.362	27.688	34.528
4	9.488	13.277	18.467	14	23.685	29.141	36.123
5	11.070	15.086	20.515	15	24.996	30.578	37.697
6	12.592	16.812	22.458	16	26.296	32.000	39.252
7	14.067	18.475	24.322	17	27.587	33.409	40.790
8	15.507	20.090	26.125	18	28.869	34.805	42.312
9	16.919	21.666	27.877	19	30.144	36.191	43.820
10	18.307	23.209	29.588	20	31.410	37.566	45.315

$\chi^2 = 6$ and Critical Value $= 7.815$

★ If $\chi^2 >$ Critical Value, then you REJECT H_0

If $\chi^2 <$ Critical Value, then you ACCEPT H_0

$6 < 7.815 \Rightarrow \chi^2 <$ Critical Value

Thus, we $\boxed{\text{Accept } H_0}$

[Calc] χ^2 Goodness of Fit Test

Correct Choice	Expected	Actual
A	25	20
B	25	20
C	25	25
D	25	35

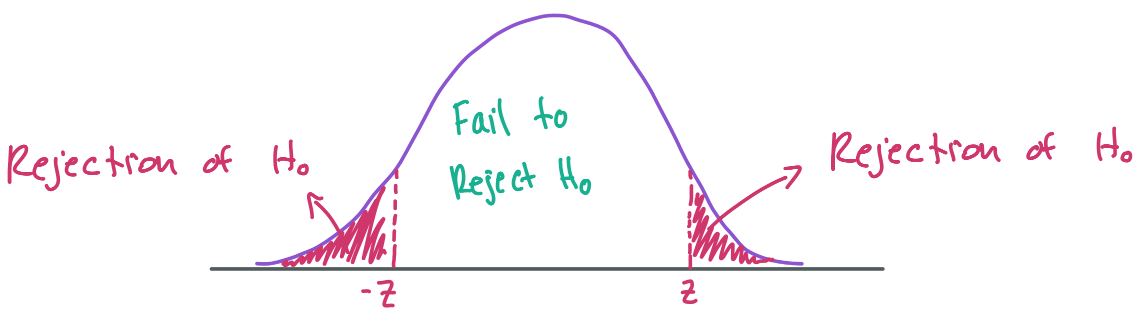
L1 → Actual Values
L2 → Expected Values

"D"

STAT → TEST → ↓ → Calculate

χ^2 GOF-Test

Critical Value (z) - Cutoff values that define regions where the χ^2 Test Statistic is unlikely to lie



- To find z, you first find your degrees of freedom (df) and choose your significance Level (α). Then use the table below

Degrees of Freedom
Critical Values
Significance Levels

Critical values of the Chi-square distribution with *d* degrees of freedom

Probability of exceeding the critical value

d	0.05	0.01	0.001	*d*	0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	7.815	11.345	16.266	13	22.362	27.688	34.528
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9	16.919	21.666	27.877	19	30.144	36.191	43.820
10	18.307	23.209	29.588	20	31.410	37.566	45.315

* You will ALWAYS be provided this table

ex)

In the game rock-paper-scissors, Kenny expects to win, tie, and lose with equal frequency. Kenny plays rock-paper-scissors often, but he suspected his own games were not following that pattern, so he took a random sample of 24 games and recorded their outcomes. Here are his results:

Outcome	Win	Loss	Tie
Games	4	13	7

He wants to use these results to carry out a χ^2 goodness-of-fit test to determine if the distribution of his outcomes disagrees with an even distribution.

What are the values of the test statistic and P-value for Kenny's test?

(A) $\chi^2 = 5.25$; $0.05 < \text{P-value} < 0.10$

(B) $\chi^2 = 5.25$; $0.15 < \text{P-value} < 0.2$

(C) $\chi^2 = 21.875$; $\text{P-value} < 0.0005$

(D) $\chi^2 = 21.875$; $0.0005 < \text{P-value} < 0.001$

H_0: even distribution of W, L, T

H_a: uneven distribution

$\alpha = 0.05$ $\quad \Sigma = 24$ $\quad$ Expected $= \frac{24}{3} = 8$

(a) $$\chi^2 = \frac{(4-8)^2}{8} + \frac{(13-8)^2}{8} + \frac{(7-8)^2}{8} \Rightarrow \boxed{\chi^2 = 5.25}$$

(b) P-Value = Probability that $\chi^2 > \underline{5.25}$

between 0.10 – 0.05

$df = 3-1 = 2$

Degree of Freedom	Probability of Exceeding the Critical Value								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09

$$\boxed{0.05 < \text{P-Value} < 0.10}$$

χ^2 Test for Independence

- Shows whether two data sets are Independent of each other or not

If P-VALUE < α, then you REJECT H_0
If P-VALUE > α, then you ACCEPT H_0

Example 2

Eighty people were asked for their favourite genre of music: pop, classical, folk or jazz. The results are in the following table.

Genre	Pop	Classical	Folk	Jazz	Totals
Male	18	9	4	7	38
Female	22	6	7	7	42
Totals	40	15	11	14	80

A χ^2 test was carried out at the 1% significance level. The critical value for this test is 11.345.

ⓐ What is H_0, H_a?

H_0: gender is independent from music preference

H_a: " not independent "

ⓑ Show that the expected value for a female liking pop is 21.

(Probability of female)$\cdot$(Probability of liking Pop)

$= \left(\frac{42}{80}\right)\left(\frac{40}{80}\right) = 26.25\%$ of total $= 0.2625 \cdot 80 = 21$

ⓒ Find the degrees of Freedom

(row $-$ 1)(column $-$ 1) $= 3 \cdot 1 = \boxed{3 \text{ df}}$

ⓓ Find χ^2 test statistic and P-Value

[Calc] χ^2 Test for Independence

Inputting Matrix

$\boxed{2nd} \rightarrow \boxed{X^{-1}} \rightarrow \boxed{EDIT} \rightarrow \underset{[A]}{\boxed{1}} \rightarrow$ Row x Column

$\rightarrow$ Enter Values using arrow key to navigate

Calculating χ^2 GOF Test

$\boxed{STAT} \rightarrow \boxed{TEST} \rightarrow \overset{\text{"C"}}{\underset{\chi^2\text{-Test}}{\boxed{\downarrow}}} \rightarrow$ Observed: edited Matrix
Expected: any empty Matrix

$\rightarrow \boxed{Calculate}$

$$\boxed{\chi^2 = 1.62 \qquad P = 0.65}$$

Clearing Matrices

$\boxed{2nd} \rightarrow \underset{\text{Mem}}{\boxed{+}} \rightarrow \underset{\text{Mem Mgt/Del}}{\boxed{2}} \rightarrow \underset{\text{Matrix}}{\boxed{5}} \rightarrow$ highlight Desired Matrix $\rightarrow \boxed{DEL}$

ⓔ Accept or Reject H_0

$P > \alpha \Rightarrow 0.65 > 0.01$ $\boxed{\text{Thus, Accept } H_0}$

Critical values of the Chi-square distribution with *d* degrees of freedom

Probability of exceeding the critical value

d	0.05	0.01	0.001	*d*	0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	7.815	11.345	16.266	13	22.362	27.688	34.528

OR

$\chi^2 < z \Rightarrow 1.62 < 11.34$ $\boxed{\text{Thus, Accept } H_0}$

χ^2 Test for Independence Ex

There hundred people of different ages were interviewed and asked which genre of film they mostly watched. The results are shown below. Using the Chi-Square Test at a 10% significance level, determine whether the genre of film watched is independent of age.

Film Type	Thriller	Comedy	Horror	Total
0-20 years	13	26	41	80
20-50 years	54	48	28	130
51+ years	39	43	8	90
Total	106	117	77	300

(a) State the null Hypothesis and Alternate Hypothesis

H_0: Independent

H_a: Not Independent

(b) Show that the expected frequency for preferring horror films between the ages of 20 and 50 years is 33.4

$P(20\text{-}50 \text{ years}) = \frac{130}{300} = 43.3\%$

$P(\text{Horror}) = \frac{77}{300} = 25\%$

$P(20\text{-}50 \text{ and Horror}) = \left(\frac{130}{300}\right)\left(\frac{77}{300}\right) \cdot 300 = 33.4$ ✓

(c) Find the degrees of Freedom

$(\# \text{ of Rows} - 1)(\# \text{ of Colums} - 1) = (2)(2) = \boxed{4}$

(d) Find χ^2 test statistic and the P-Value

$\boxed{\chi^2 = 45.2}$ $\boxed{\text{P-VALUE} = 3.5e^{-9}}$

(e) Comment on your Result

$\text{P-VALUE} < \alpha \Rightarrow 3.9e^{-9} < 0.1$ $\boxed{\text{Thus, we REJECT } H_0}$

$\chi^2 > z \Rightarrow 45.2 > 7.78$ $\boxed{\text{Not Independent}}$

χ^2 Goodness of fit ex

Example 4

The students in Year 8 are asked what day of the week their birthdays are on this year. The table shows the results.

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Frequency	12	14	18	17	15	15	14

a Write down the table of expected values, given that each day is equally likely.

b Conduct a χ^2 goodness of fit test at the 5% significance level for this data.

c The critical value is 12.592. Write down the conclusion for the test.

a

Day	Expected	Actual
Sunday	15	12
Monday	15	14
Tuesday	15	18
Wednesday	15	17
Thursday	15	15
Friday	15	15
Saturday	15	14

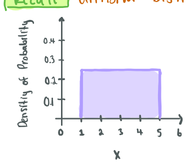
Σ students $= 105/7 = 15$

b H_0: even / uniform distribution

H_a: uneven distribution

$\alpha = 0.05$ $\quad$ $df = 7 - 1 = 6$

Recall Uniform Distribution



L1 → Actual + L2 → Expected

$\boxed{\text{STAT}} \rightarrow \boxed{\text{TEST}} \rightarrow \boxed{\downarrow} \rightarrow \boxed{\text{Calculate}}$ → $\boxed{\chi^2 = 1.6}$

χ^2 GOF - Test

c $\left.\begin{array}{l} z = 12.592 \\ \chi^2 = 1.6 \end{array}\right\} \Rightarrow \chi^2 < z$ $\quad$ Thus, we Accept H_0

OR

$\left.\begin{array}{l} \text{P-Value} = 0.95 \\ \alpha = 0.05 \end{array}\right\} \Rightarrow \text{P-Value} > \alpha$ $\quad$ Thus, we Accept H_0

Even Distribution